SSLC

SIMILAR TRIANGLES

ENLISH VERSION

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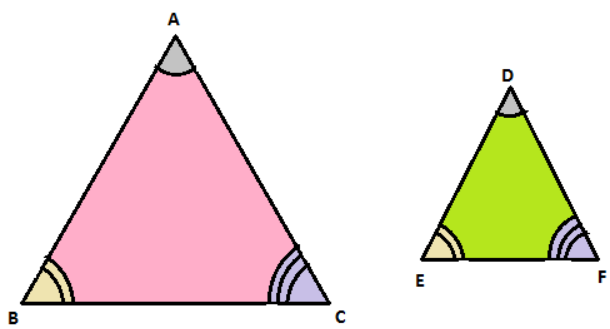
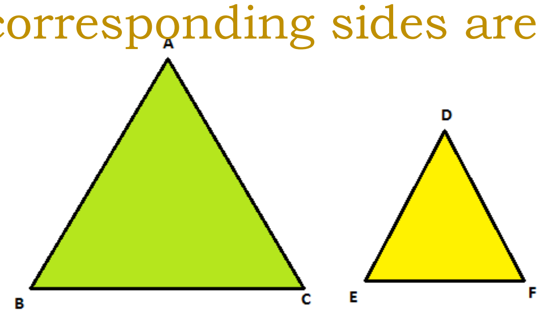
Chapter 11

Similar Triangles

- Two triangles are said to be similar, if
  - Their corresponding angles are equal.



  - Their corresponding sides are proportional



$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

Thales Theorem :[Basic proportionality theorem]

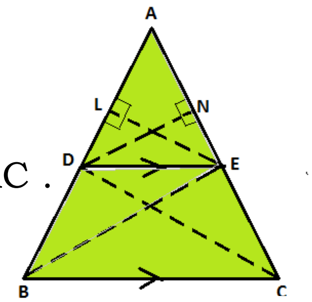
"If a straight line is drawn parallel to a side of a triangle, then it divides the other two sides proportionally"

Data : In ΔABC, $DE \parallel BC$

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : 1. Join D,E and E,B .

2. draw $EL \perp AB$ and $DN \perp AC$.



Proof :

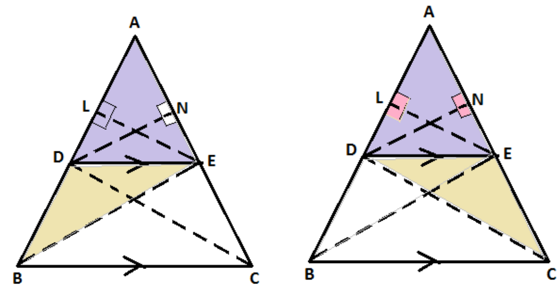
$\frac{\Delta ABC}{\Delta BDE} = \frac{\frac{1}{2} xADxEL}{\frac{1}{2} xDBxEL}$ $\because A = \frac{1}{2}xbxh$

$\frac{\Delta ABC}{\Delta BDE} = \frac{AD}{DB}$

$\frac{\Delta ADE}{\Delta CDE} = \frac{\frac{1}{2} xAExDN}{\frac{1}{2} xDBxDN}$ $\because A = \frac{1}{2}xbxh$

$\frac{\Delta ADE}{\Delta CDE} = \frac{AE}{EC}$

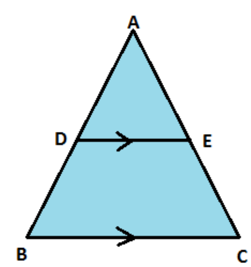
$\frac{AD}{DB} = \frac{AE}{EC}$ $\because \Delta BDE \equiv \Delta CDE$



- Corollary;

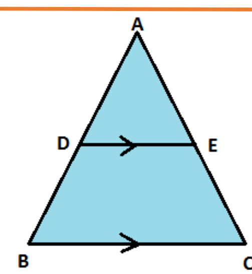
1. In ΔABC DE∥BC,

$\frac{AB}{DB} = \frac{AC}{EC}$



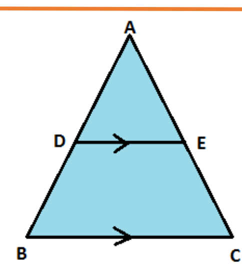
2. In ΔABC DE∥BC,

$\frac{AB}{AD} = \frac{AC}{AE}$



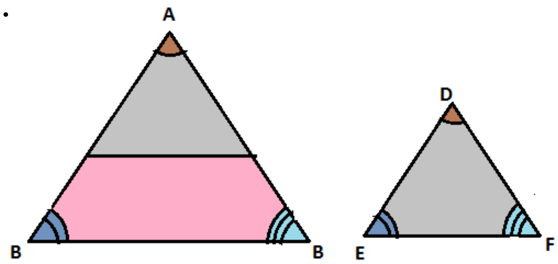
3. In ΔABC DE∥BC,

$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$



Theorem (AA similarity Criterion)

"If two triangles are equiangular , then their corresponding sides are proportional" .



Data :

In ΔABC ಮತ್ತು ΔDEF

(i). ∠BAC = ∠EDF

(ii). ∠ABC = ∠DEF

To prove : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

Construction :

Mark points 'G' and 'H' on AB and AC such that

i) AG = DE and

ii) ii) AH = DF join G and H.

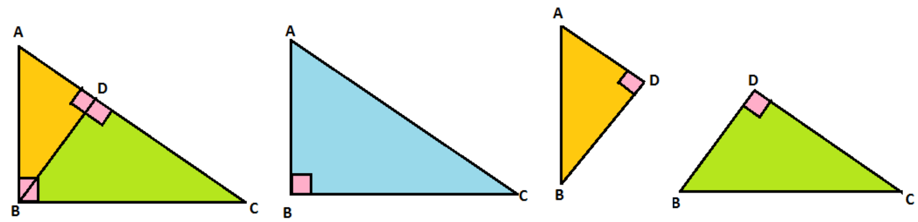
Proof :

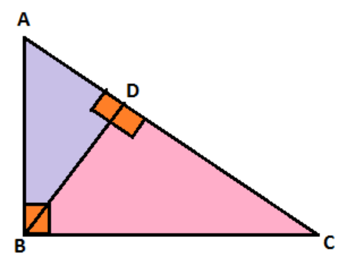
In ΔAGH and ΔDEF

AG = DE	∵ construction
∠BAC = ∠EDF	∵ data
AH = DF	∵ construction
∴ ΔAGH ≡ ΔDEF	∵ SAS
∴ ∠AGH = ∠DEF	∵ CPCT
ಆದರೆ∠ABC = ∠DEF	∵ Data
⇒ ∠AGH = ∠ABC	∵ Axiom 1
∴ GH ∥ BC	
∴ $\frac{AB}{AG} = \frac{BC}{GH} = \frac{CA}{HA}$	∵ third corollary to Thales theorem
∴ $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$	∵ ΔAGH ≡ ΔDEF

THEOREM.

"In a right angled triangle, the perpendicular to the hypotenuse from the right angled vertex, divides the original triangle into two right angled triangles, each of which is similar to the original triangle."





Data : In

ΔABC ,(i) $\angle ABC = 90^0$ (ii) $BD \perp AC$

To prove:

(i) $\Delta ADB \sim \Delta ABC$

(ii). $\Delta BDC \sim \Delta ABC$

(iii). $\Delta ADB \sim \Delta BDC$

Proof : compare ΔADB and ΔABC ,

(i). $\angle ADB = \angle ABC = 90^0$	$\because$ data
(ii). $\angle BAD = \angle CAD$	$\because$ common angle
(iii). $\angle ABD = \angle ACB$	$\because$ Δ sum of three angles of a Δ is 180^0
$\therefore$ $\Delta ADB \sim \Delta ABC$ ....(1)	$\because$ Equiangular triangles
In ΔBDC and ΔABC ,	
(i).$\angle BDC = \angle ABC = 90^0$	$\because$ data
(ii). $\angle BCD = \angle ACB$	$\because$ common angle
(iii). $\angle DBC = \angle BAC$	Δ sum of three angles of a Δ is 180^0]
$\therefore$ $\Delta BDC \sim \Delta ABC$ ....(2)	$\because$Equiangular triangles]

From (1) ಮತ್ತು (2)

$\Delta ADB \sim \Delta BDC$

Corollary-1

$\triangle$ ADB ~ $\triangle$ ABC

$AB^2 = AC.AD$

Corollary – 2

$\triangle$ BDC ~ $\triangle$ ABC

$BC^2 = AC.DC$

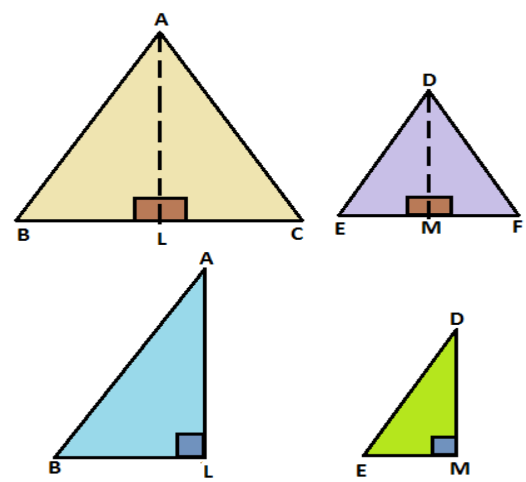
Corollary – 3

$\triangle$ ADB ~ $\triangle$ BDC

$BD^2 = AD.DC$

Theorem

"The areas of similar triangles are proportional to squares on the corresponding sides".



Data :

$\Delta ABC \sim \Delta DEF$,

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{DF}$$

To prove :

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{BC^2}{EF^2}$$

Construction :

Draw $AL \perp BC$ and $DM \perp EF$

Proof : compare ΔALB and ΔDME

$\angle ABL = \angle DEM$ [∵ data]

$\angle ALB = \angle DME = 90^0$ [∵ construction]

$\Delta ALB \sim \Delta DME$ [∵Equiangular]

$\Rightarrow \frac{AL}{DM} = \frac{AB}{DE}$ ಆದರೆ $\frac{BC}{EF} = \frac{AB}{DE}$ [∵ data]

$\therefore \frac{AL}{DM} = \frac{BC}{EF}$ ......(1)

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{\frac{1}{2} \text{x} BC \text{x} AL}{\frac{1}{2} \text{x} EF \text{x} DM}$$

$$= \frac{BC \text{x} AL}{EF \text{x} DM} \quad [\because (1)]$$

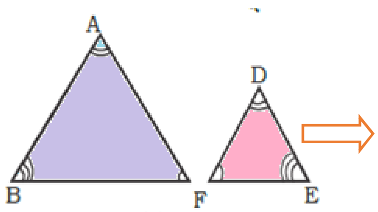
$$= \frac{BC \text{x} BC}{EF \text{x} EF} = \frac{BC^2}{EF^2}$$

From data, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{DF}$

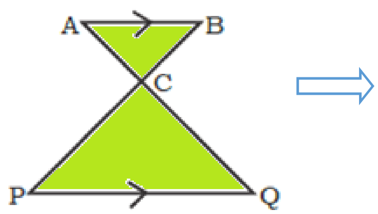
$$\therefore \frac{\text{Area of } \Delta ABc}{\text{Area of } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{CA^2}{DF^2}$$

Exercise : 11.1

1) In the given pairs of similar triangles, write the corresponding vertices, corresponding sides and their ratios.

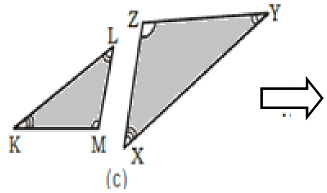


(a)



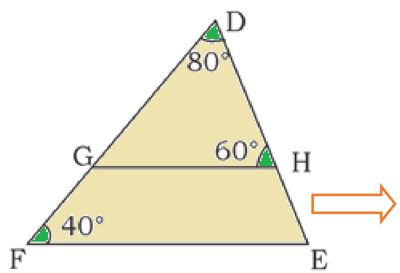
(b)

Correspondi ng vertices	Correspondi ng sides	Ratios
A → D **B → E** **C → F**	AB → DE BC → EF AC → FD	$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{FD}$
A → Q **B → P** **C → C**	AB → PQ BC → PC AC → CQ	$\frac{AB}{PQ} = \frac{BC}{PC} = \frac{AC}{CQ}$



L → Y **M → Z** **K → X**	ML → YZ MK → XZ KL → XY	$\frac{ML}{YZ} = \frac{MK}{XZ} = \frac{KL}{XY}$

2) Study the following figures and find out in each case whether the triangles are similar. Give reason.



Solution: In ΔDGH

$\angle G = 180° - (60° + 80°)$

$= 180° - 140° = 40°$

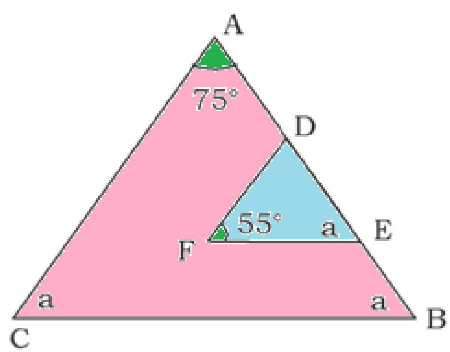
In ΔDFE

$\angle E = 180° - (40° + 80°)$

$= 180° - 120° = 60°$

$\angle G = \angle F, \angle D = \angle D, \angle H = \angle E$

$\therefore \Delta DGH \sim \Delta DFE$



Solution : In ΔABC

$\angle B = \angle C = a$

$75° + a + a = 180°$

$75° + 2a = 180°$

$2a = 180° - 75°$

$2a = 105°$

$a = \frac{105°}{2} = 52.5°$

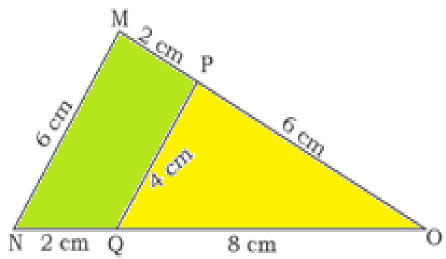
$\angle B = \angle C = 52.5° \therefore \angle E = 52.5°$

ΔDFEಯಲ್ಲಿ $\angle D = 180° - (55° + 52.5°)$

$\angle D = 180° - 107.5° = 72.5°$

Corresponding angles of triangles are not equal.

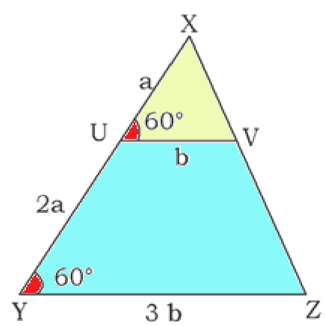
$\therefore \Delta ABC \sim \Delta DFE$



Solution : In ΔMON and ΔPOQ

$$\frac{MO}{PO} = \frac{8}{6} = \frac{4}{3}$$
$$\frac{MN}{PQ} = \frac{6}{4} = \frac{3}{2}$$
$$\frac{NO}{OQ} = \frac{10}{8} = \frac{5}{4}$$
$$\frac{MO}{PO} \neq \frac{MN}{PQ} \neq \frac{NO}{OQ}$$

$\therefore$ ΔMON and ΔPOQ are not similar triangles.



soln : : In ΔXUV and ΔXYZ ,

$\angle XUV = \angle XYZ = 60°$

$\angle X$ common angle

$\angle XVU = \angle XZY$ [corresponding angles]

Corresponding angles of triangles are equal

$\therefore \Delta XUV \sim \Delta XYZ$

3. From the following data, state whether ΔABC is similar to ΔDEF or not

a)$\angle A = 70°, \angle B = 80°, \angle D = 70°, \angle F = 30°$

Soln:

$\angle C = 180° - (70° + 80°) = 180° - 150° = 30°$

$\angle E = 180° - (70° + 30°) = 180° - 100° =$

$80°$corresponding angles of triangles are equal

$\therefore \Delta ABC \sim \Delta DEF$

b)$AB = 8cm, BC = 9cm, CA = 15cm, DE = 4cm, EF = 3cm, FD = 5cm$

soln :

$\frac{AB}{DE} = \frac{8}{4} = 2$

$\frac{BC}{EF} = \frac{9}{3} = 3$

$\frac{CA}{FD} = \frac{15}{5} = 3$

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = 3$

Corresponding sides of triangles are not proportional

$\therefore$ ΔABCಮತ್ತು ΔDEF are not similar

4. Select the set of numbers in the following, which can form similar triangles.

(i) 3,4,6 (ii) 9,12,18 (iii) 8,6,12 (iv) 8,4,9 (v) $2, 4\frac{1}{2}, 4$

a) (i) ಮತ್ತು (ii) $\rightarrow \frac{3}{9} = \frac{4}{12} = \frac{6}{18} = \frac{1}{3}$

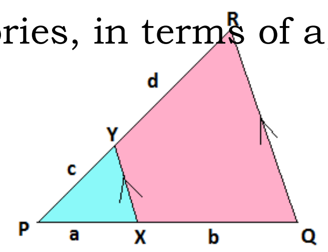
b) (i) ಮತ್ತು (iii) $\rightarrow \frac{3}{6} = \frac{4}{8} = \frac{6}{12} = \frac{1}{2}$

c) (ii) ಮತ್ತು (iii) $\rightarrow \frac{9}{6} = \frac{12}{8} = \frac{18}{12} = \frac{3}{2}$

Can form similar triangles.

Exercise 10.2

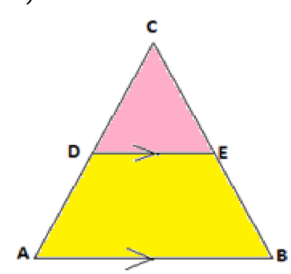
1. Study the adjoining figure. Write the ratios in relation to basic proportionality theorem and its cocollories, in terms of a, b, c, and d



Soln ; $\frac{a+b}{a} = \frac{c+d}{c}$

2. In the adjoining figure, DE∥AB, AD = 7cm, CD = 5cm and BC = 18cm.

Find BE and CE .



Soln : In the figure DE∥AB,

According to B.P,T,

$\frac{AC}{CD} = \frac{BC}{CE}$

$\frac{12}{5} = \frac{18}{CE}$

$CE = \frac{18x5}{12}$

$CE = 7.5cm$

∴ BE = BC – CE

∴ BE = 18 – 7.5

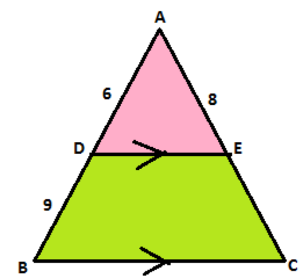
∴ BE = 10.5cm

3.In ΔABC,D and E are points on the sides AB and AC respectively such that DE∥BC.

i) If. AD = 6cm,DB = 9cm and AE = 8cm find AC..

Soln: In the fig DE∥BC so

according to B.P.T,



$$\frac{AB}{AD} = \frac{AC}{AE}$$

$$AC = \frac{AB x AE}{AD}$$

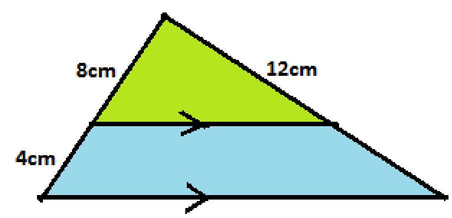
$$AC = \frac{15x8}{6}$$

$$AC = 20cm$$

(ii) AD = 8cm,AB = 12cm and AE = 12cm Find CE..

soln :In the fig DE∥BC so

according to B.P.T,



$$\frac{AB}{AD} = \frac{AC}{AE}$$

$$AC = \frac{AB x AE}{AD}$$

$$AC = \frac{12x12}{8}$$

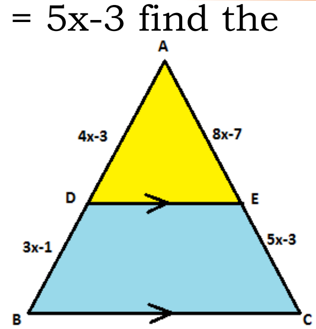
$$AC = 18cm$$

∴ CE = AC – AE = 18 – 12 = 6cm

(iii) AD = 4x-3 ,BD = 3x-1, AE = 8x -7 and CE = 5x-3 find the value of x.

soln: In the fig DE∥BC so

according to B.P.T,



$$\frac{AD}{BD} = \frac{AE}{CE}$$

$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

$(4x-3)(5x-3) = (8x-7)(3x-1)$

$20x^2 - 15x - 12x + 9 = 24x^2 - 21x - 8x + 7$

$-4x^2 + 2x + 2 = 0$

$4x^2 - 2x - 2 = 0$

$4x^2 - 4x + 2x - 2 = 0$

$4x(x - 1) + 2(x - 1) = 0$

$(x - 1)(4x + 2) = 0$

$(x - 1) = 0$ or $(4x + 2) = 0$

X = 1 or 4x = - 2 (length of line cannot be positive)

$\therefore$ X = 1

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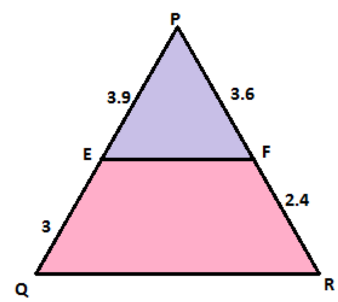
3) In ΔPQR, E and F are points on the sides PQ and PR, respectively. For each of the following cases, verify EF∥QR.

(i). PE = 3.9cm, EQ = 3cm, PF = 3.6cm, FR = 2.4cm



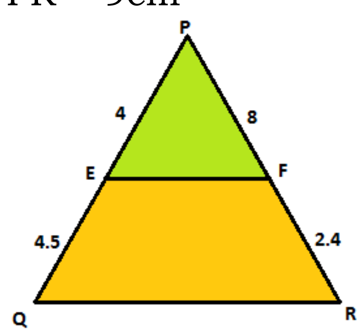
$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$

$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$

$\therefore \frac{PE}{EQ} \neq \frac{PF}{FR}$

$\therefore$ EF≠QR .

(ii) PE = 4cm, QE = 4.5cm, PF = 3.6cm, FR = 9cm



$\frac{PE}{EQ} = \frac{4}{4.5} = 0.889$

$\frac{PF}{FR} = \frac{8}{9} = 0.889$

$\therefore \frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore$ EF∥QR

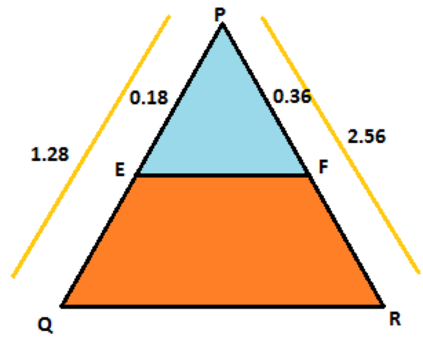
(iii) PQ = 1.28cm, PR = 2.56cm, PE = 0.18cm, PF = 0.36cm

$\frac{PQ}{PE} = \frac{1.28}{0.18} = 7.11$

$\frac{PR}{PF} = \frac{2.56}{0.36} = 7.11$

$\therefore \frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore EF \parallel QR$



5. In the adjoining figure, AC∥BD and CE∥DF .If OA = 12cm,AB = 9cm, OC = 8cm and EF = 4.5cm, find OE..

soln ; In ΔOBD, AC∥BD

$\therefore \frac{OA}{AB} = \frac{OC}{CD}$

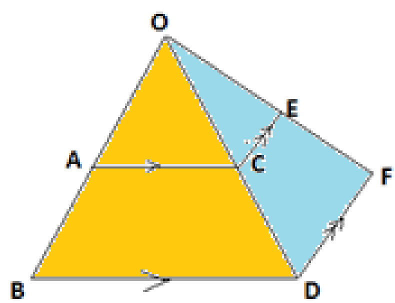
$\Rightarrow \frac{12}{9} = \frac{8}{CD}$

$\Rightarrow CD = \frac{8x9}{12}$=6cm

Δ ODF ನಲ್ಲಿ CE∥DF

$\therefore \frac{OC}{CD} = \frac{OE}{EF}$

$\Rightarrow OE = \frac{8x4.5}{6}$ =6cm



6. In the figure PC∥QK and BC∥HK. If AQ = 6cm,QH = 4cm, HP = 5cm and KC = 18cm, find AK and PB.

Soln: In ΔAPC, QK∥PC

$\frac{AQ}{QP} = \frac{AK}{KC}$

$\Rightarrow \frac{6}{9} = \frac{AK}{18}$

$\Rightarrow AK = \frac{18x6}{9} = 12cm$

In ABC ,HK∥BC

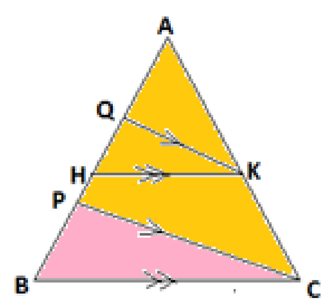
$\therefore \frac{AH}{HB} = \frac{AK}{KC}$

$\Rightarrow \frac{10}{HB} = \frac{12}{18}$

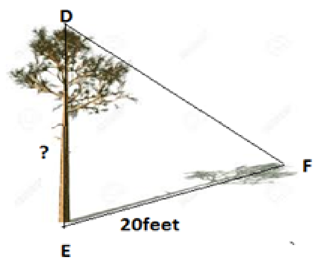
$\Rightarrow HB = \frac{18x10}{12} = 15cm$

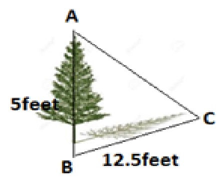
∴ PB = HB - HP

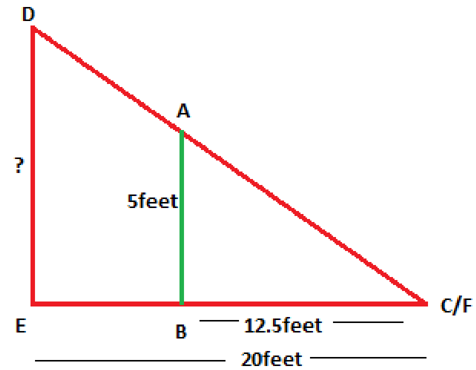
⇒ PB = 15 – 5 = 10cm



7) At a certain time of the day a tree casts its shadow 12.5 feet long. If the height of the tree is 5 feet, find the height of another tree that casts its shadow 20 feet long at the same time.







In the fig AB∥DE,

$\therefore \frac{DE}{AB} = \frac{EF}{BC} \Rightarrow \frac{DE}{5} = \frac{20}{12.5} \Rightarrow DE = \frac{20x5}{12.5} = 8$ feet

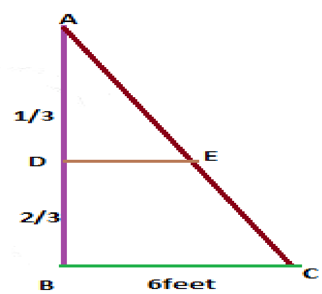
8) A lader resting against a vertical wall has its foot on the ground at a distance of 6cm from the wall. Aman on the ground climbes two thirds of the ladder. What will be his distance from the wall now?.

Soln: In fig DE∥BC,

$\therefore \frac{DE}{BC} = \frac{AD}{AB} \Rightarrow DE = \frac{ADxBC}{AB}$

$\Rightarrow DE = \frac{\frac{1}{3}x6}{1}$

$\Rightarrow DE = 2$feet

Riders based on Thales theorem

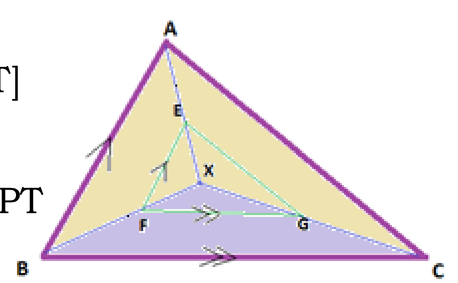
1. 'X' is any point inside ΔABC. XA,XB and XC are joined. 'E' is any point on $\overline{AX}$. If EF∥AB, FG∥BC.Prove that EG∥AC

soln: In ΔAXB , EF∥AB

$\therefore \frac{XE}{EA} = \frac{XF}{FB}$----------(1) [∵ by using BPT]

In ΔBXC , FG∥BC

$\therefore \frac{XF}{FB} = \frac{XG}{GC}$----------(2) [∵ ∵ by using BPT

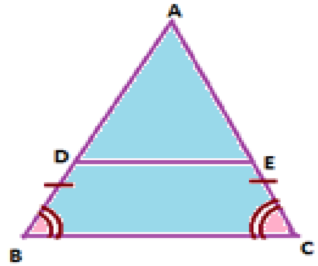


From (1) and (2)

$\frac{XE}{EA} = \frac{XG}{GC}$

∴ EG∥AC [∵ by using converse of BPT]

2. In ΔAXB , D and E are points on AB and AC such that BD = CE. Prove that DE∥BC.



Soln ; In ΔABC, ∠B = ∠C

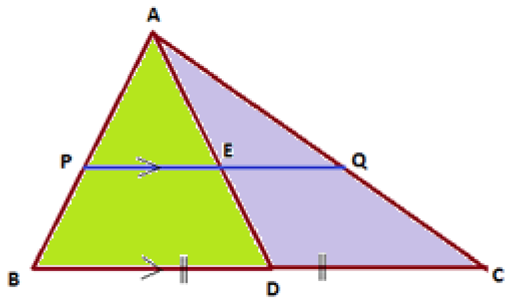
∴ AB = AC

BD = EC [∵ data]

∴ $\frac{AB}{BD} = \frac{AC}{EC}$

∴ DE∥BC [∵ by using converse of BPT]

3. In ΔABC, PQ∥BC and BD = DC .Prove that PE = EQ .



Soln ; In ΔABD, PE∥BD [∵ data]

∴ $\frac{AE}{AD} = \frac{PE}{BD}$ ------ (1) [∵ B.P.T]

In ΔADC, EQ∥DC [∵ data]

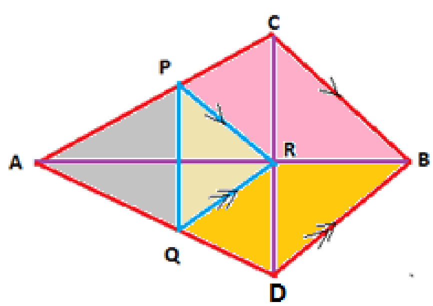
∴ $\frac{AE}{AD} = \frac{EQ}{DC}$ ------ (2) [∵B.P.T]

From (1) And (2)

$\frac{PE}{BD} = \frac{EQ}{DC}$

PE = EQ [∵ BD = DC data]

4. In the figure PR∥BC and QR∥BD. Prove that PQ∥CD .



Soln : In ΔABC, PR∥BC

∴ $\frac{AR}{RB} = \frac{AP}{PC}$ ------ (1) [∵ B.P.T]

In ΔADB, QR∥BD

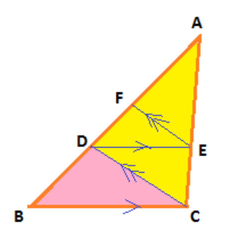
∴ $\frac{AR}{RB} = \frac{AQ}{QD}$ ------ (2) [∵ B.P.T]

From (1) and (2)

$\frac{AP}{PC} = \frac{AQ}{QD}$

∴ PQ∥CD [∵ by using converse of BPT]

5. In ΔABC , DE∥BC and CD∥EF. Prove that .AD^2 = AF x AB .



Soln : In ΔAB, DE∥BC

∴ $\frac{AB}{AD} = \frac{AC}{AE}$ ------ (1) [∵ B.P.T]

In ΔADC , FE∥DC

∴ $\frac{AD}{AF} = \frac{AC}{AE}$ ------ (2) [∵ B.P.T]

From (1) and (2)

$\frac{AB}{AD} = \frac{AD}{AF}$

∴ ADxAD = AB x AF

$AD^2 = AF \times AB$

Exercise11.3

A. Numerical problems based on AA similarity criterion.

1. In the given figure, AE∥DB, BC = 7cm, BD = 5cm, DC = 4cm, if CE = 12cm , find AE and AC.

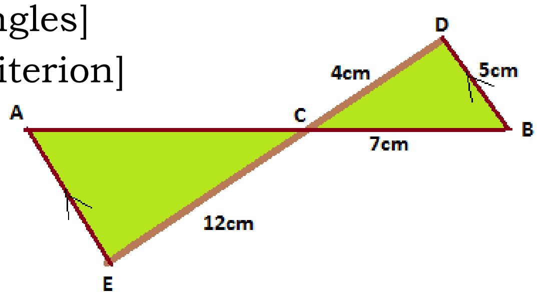
   Soln :

   In ΔACE and ΔBDC,

   Δ*AA*ACE = ΔBCD [∵vertically opp angles]

   ∠EAC = ∠DBC [∵alternate angles]

   ∴ ΔACE ~ ΔBDC [∵by AAA Criterion]



   $\therefore \frac{AC}{BC} = \frac{CE}{CD}$

   $\Rightarrow AC = \frac{12x7}{4} = 21cm$

   $\Rightarrow AE = \frac{12x5}{4} = 15cm$

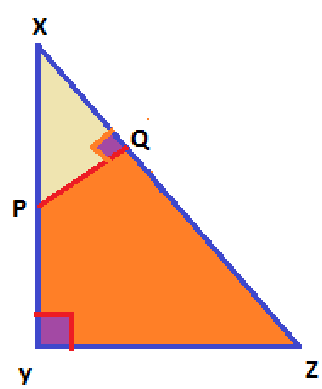
2. In ΔXYZ,Pis any point on XY and PQ⊥XZ.If XP = 4cm XY = 16cm and XZ = 24cm, find XQ.

   Soln ; In ΔXYZ and ΔXQP,

   ∠XYZ = ∠XQP [∵90^0]

   ∠YXZ = ∠QXP [∵common angle]

   ∴ ΔXYZ ~ Δ XQP [∵by AAA Criterion]



   $\therefore \frac{XQ}{XY} = \frac{XP}{XZ}$

   $\Rightarrow XQ = \frac{XPxXY}{XZ}$

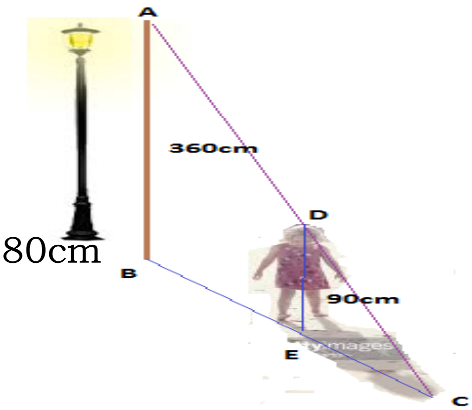
   $\Rightarrow XQ = \frac{4x16}{24}$

   ⇒ XQ = 2.6cm

3. A girl of height 90cm is walking away from the bas of a lamp-postat a speed of 1.2m/s. If the lamp is 3.6m above the ground, find the length of her shadow after 4 seconds.

speed of the girl = 120 cm/s

4s ಗಳಲ್ಲಿ ನಡೆದ ದೂರ = 120x4 = 480cm



Height of lamp = AB = 360cm

Height of girl = DE = 90cm

Distance walked by the girl = BE = 480cm

In ΔABC and ΔDEC,

$\angle ABC = \angle DEC = 90^0$

$\angle C = \angle C$ [∵common angle]

∴ Δ ABC ~ Δ DEC [∵by AAA Criterion]

$\therefore \frac{EC}{BC} = \frac{DE}{AB}$

$\therefore \frac{EC}{BE+EC} = \frac{DE}{AB}$

$\therefore EC = \frac{DE[BE+EC]}{AB}$

$\therefore EC = \frac{90[480+EC]}{360}$

$\therefore EC = \frac{[480+EC]}{4}$

$\therefore 4EC = 480 + EC$

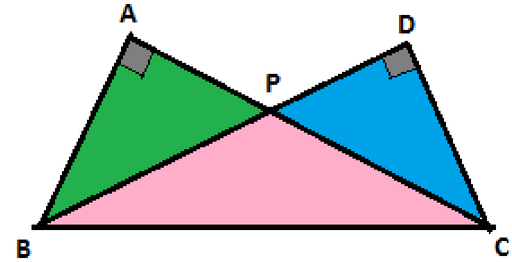
$\therefore 3EC = 480$

$\therefore EC = 160cm$

∴length of shadow = EC = 1.6m

Rriders based on AA similarity criterion

1. ΔBAC and ΔBDC are two right angled triangles with common hypotenuse BC. The sides AC and BD intersect at P.Prove that AP.PC = DP.PB .



Soln : In ΔABP and ΔDCP,

∠A = ∠D = 90^0

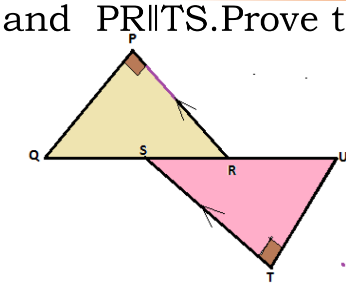
∠APB = ∠DPC [∵vertically opp angles]

∴ ΔABP ~ ΔDCP

∴ $\frac{AP}{DP} = \frac{PB}{PC}$ [corresponding sides of similar triangles are proportional]

⇒ AP.PC = DP.PB

2. In ΔPQR and Δ TUS , ∠QPR = ∠UTS = 90^0 and PR∥TS.Prove that ⊿ PQR ~ ⊿TUS.



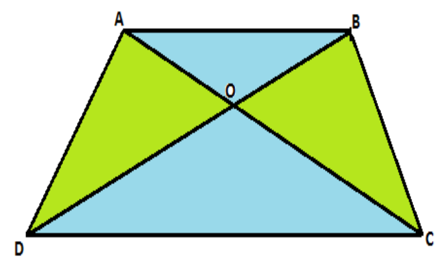
Soln : In ΔQPR and ΔTUS

$\angle P = \angle T = 90^0$

$\angle R = \angle S$ [∵alternate angles]

ΔPQR ~ Δ TUS [∵by AAA Criterion]

3. If the diagonals of a quadrilateral divide each other proportionally , then Prove that the quadrilateral ABCD is a trapezium,



$\frac{AO}{OC} = \frac{OB}{OD}$ [∵ data]

⇒ ΔAOB ~ ΔCOD

∴ ∠0AB = ∠0CD

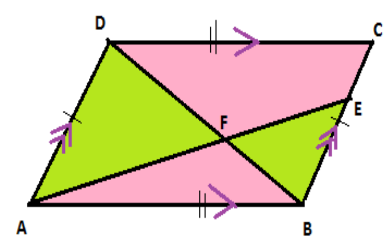
But they are alternate angles

∴ AB ∥ CD

∴ Hence, the quabrilateral ABCD trapezium

4. The diagonal BD of a parallelogram ABCD intersect AE at 'F'. E is any point on BC.

Prove that DF x EF = FB x FA.



Soln ; In ΔAFD and ΔBFE

∠AFD = ∠BFE [∵vertically opp angles]

∠ADF = ∠EBF [∵alternate angles]

ΔAFD ~ Δ BFE [∵by AAA Criterion]

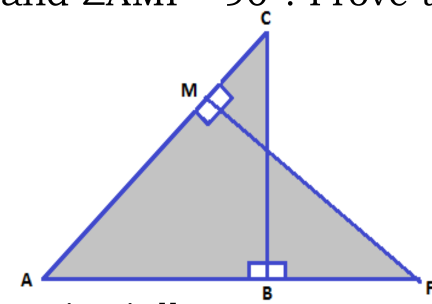
$\therefore \frac{DF}{FB} = \frac{FA}{EF}$

$\Rightarrow DF.EF = FB.FA$

5. In the adjoining figure, $\angle ABC = 90^0$ and $\angle AMP = 90^0$. Prove that

(i). $\triangle ABC \sim \triangle AMP$

(ii). $\frac{CA}{PA} = \frac{BC}{MP}$



Soln: In ΔABC and ΔPMA,

$\angle ABC = \angle AMP = 90^0$ [$\because$ data]

$\angle BAC = \angle MAP$ [$\because$common angle]

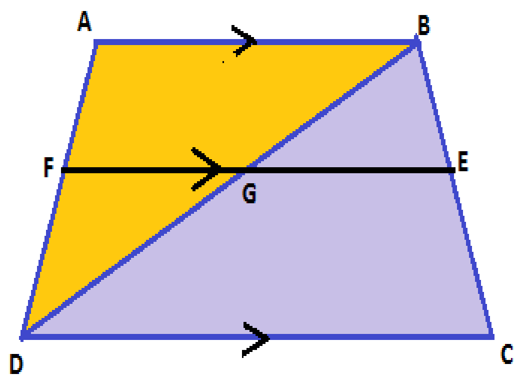
(i). $\Delta ABC \sim \Delta AMP$ [$\because$AAA similarity criteria]]

(ii). $\frac{CA}{PA} = \frac{BC}{MP}$

6. In the trapezium ABCD, AB∥DC, EF∥AB, DC = 2AB ಮತ್ತು $\frac{BE}{EC} = \frac{3}{4}$,

Prove that 7EF = 10AB.

soln: In ΔABD, FG∥AB,



$\frac{BE}{EC} = \frac{3}{4} \Rightarrow \frac{EC}{BE} = \frac{4}{3} \Rightarrow \frac{BC-BE}{BE} = \frac{4}{3}$

$\Rightarrow \frac{BC}{BE} - \frac{BE}{BE} = \frac{4}{3} \Rightarrow \frac{BC}{BE} - 1 = \frac{4}{3}$

$\Rightarrow \frac{BC}{BE} = \frac{4}{3} + 1 = \frac{7}{3}$

$\Rightarrow \frac{BE}{BC} = \frac{3}{7}$ --------- --- (1)

$\therefore \frac{FG}{AB} = \frac{GD}{BD}$

$\Rightarrow \frac{FG}{AB} = \frac{BD-BG}{BD} \Rightarrow \frac{BD}{BD} - \frac{BG}{BD}$

$\Rightarrow \frac{FG}{AB} = 1 - \frac{BG}{BD}$ --------- (2)

In ΔBDC, GE∥DC,

$\therefore \frac{GE}{DC} = \frac{BG}{BD} = \frac{BE}{BC}$ **------- (3)**

(1)$\Rightarrow \frac{FG}{AB} = 1 - \frac{BG}{BD} \Rightarrow 1 - \frac{BE}{BC} \Rightarrow 1 - \frac{3}{7}$ [$\because \frac{BE}{BC} = \frac{3}{7}$]

$\Rightarrow \frac{FG}{AB} = \frac{4}{7}$

$\Rightarrow$ 7FG = 4AB -------- (4)

And $\therefore \frac{GE}{DC} = \frac{BE}{BC}$ [$\because$ from (3)]

$\Rightarrow \frac{GE}{2AB} = \frac{3}{7}$ [∵ (1)and DC = 2AB]

$\Rightarrow$ **7GE = 6AB -------(5)**

(4)+ (5)

7FG + 7GE = 4AB + 6AB

= 7(FG + GE) = 10AB

= 7EF = 10AB [∵FG + GE = EF]

Exercise 11.4

1. In which of the following cases the pairs of triangles are similar? Write the similarity criterion used by you for answering the questions and also write the pair of similar triangles in the symbolic form.

Fig – 1

$\angle A = \angle D$

$\angle C = \angle F$

∴ ΔABC ~ Δ DEF [∵AAA similar criteria]

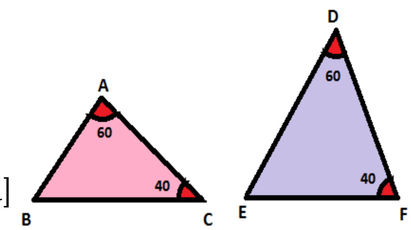
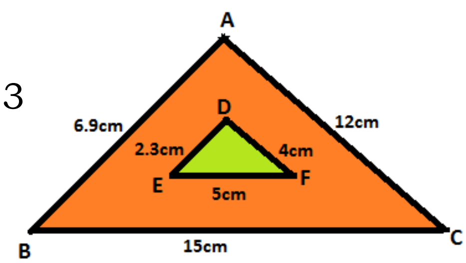


Fig – 2

$\frac{AB}{DE} = \frac{6.9}{2.3} = 3; \frac{AC}{DF} = \frac{12}{4} = 3; \frac{BC}{EF} = \frac{15}{5} = 3$

$\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = 3$

∴ ΔABC ~ Δ DEF [∵B.P.T]



Fig– 3

$\frac{OW}{OY} = \frac{7}{4}$ and $\frac{OX}{OY} = \frac{7}{4}$

∴ ΔWOX ~ Δ ZOY [∵B.P.T.]

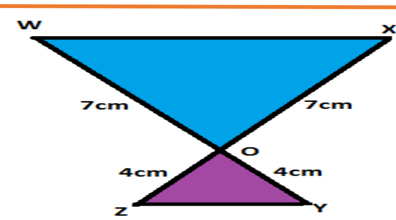
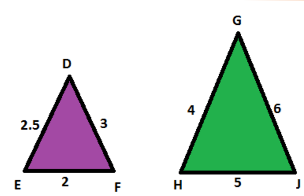


Fig – 4

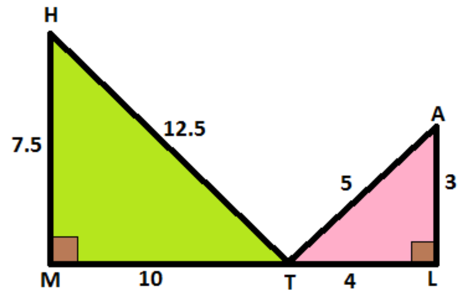
$\frac{HJ}{DE} = \frac{5}{2.5} = 2; \frac{GJ}{DF} = \frac{6}{3} = 2; \frac{GH}{EF} = \frac{4}{2} = 2$

$\therefore \frac{HJ}{DE} = \frac{GJ}{DF} = \frac{GH}{EF} = 2$



∴ $\Delta GHJ \sim \Delta DEF$ [∵B.P.T.]

Fig - 5



$\frac{HT}{AT} = \frac{12.5}{5} = 2.5$; $\frac{HM}{AL} = \frac{7.5}{3} = 2.5$;

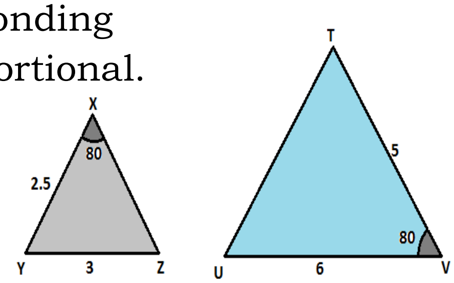
$\frac{MT}{LT} = \frac{10}{4} = 2.5$

∴ $\frac{HT}{AT} = \frac{HM}{AL} = \frac{MT}{LT} = 2.5$

∴ **ΔHMT ~ Δ ALT [∵B.P.T.]**

Fig – 6

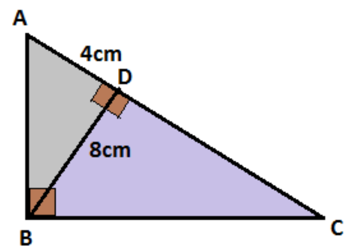
In ΔABC and ΔUTV, the sides corresponding to the equal angle (80^0) are Not proportional. So, they are not similar.



Exercise 11.5

1. In Δ ABC , $\angle ABC = 90^0$, $BD \perp AC$

(a) BD = 8cm, AD = 4cm , Find CD



$BD^2 = AD \times CD$

$8^2 = 4 \times CD$

$64 = 4CD$

∴ CD = 16cm

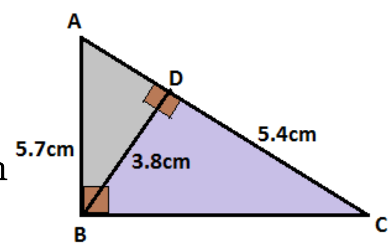
(b) AB = 5.7cm, BD = 3.8cm, CD = 5.4cm find BC.

$BD^2 = AD \times CD$

∴ $3.8^2 = AD \times 5.4$

$AD = \frac{14.44}{5.4} = 2.67cm$

∴ AC = AD + CD = 2.67 + 5.4 = 8.07cm

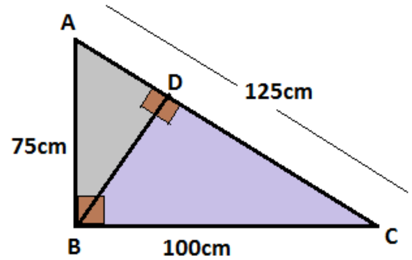


$BC^2 = AC \times CD$

$BC^2 = 8.07 \times 5.4 = 43.6$

BC = 6.6cm

(c). AB = 75cm, BC = 1m , AC = 1.25m, find BD.



$AB^2 = AC \times AD$

$75^2 = 125 \times AD$

$AD = \frac{5625}{125} = 45cm$

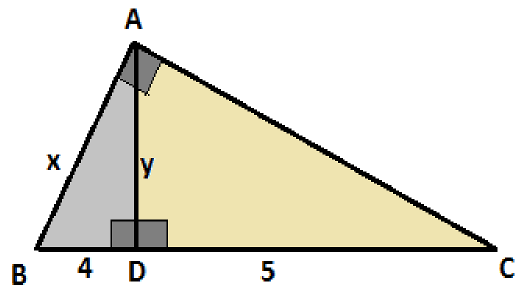
$BD^2 = AD \times CD$

$BD^2 = 45 \times 80 = 3600$

BD = 60cm

2) In Δ ABC, $\angle BAC = 90^0$, $AD \perp BC$, BD = 4cm, DC = 5cm

Find x and y.



$AD^2 = BD \times CD$

$y^2 = 4 \times 5 = 20$

$y = \sqrt{20}$

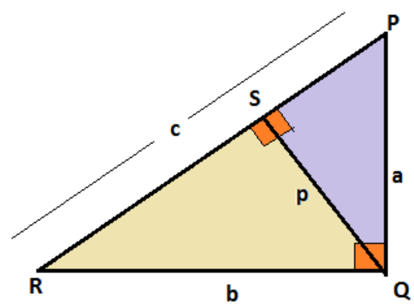
$y = 2\sqrt{5}cm$ or 4.47cm

$AB^2 = BC \times BD$

$x^2 = 9 \times 4 = 36$

x = 6cm

3. In Δ PQR, $\angle PQR = 90^0$, $QS \perp PR$, PQ = a, QR = b, RP = c and QS = p, show that pc = ab



Soln : $QR^2 = RP \times SR \Rightarrow b^2 = c \times SR$

$SR = \frac{b^2}{c}$

$PQ^2 = RP \times SP \Rightarrow a^2 = c \times SP$

$SP = \frac{a^2}{c}$

$SQ^2 = SR \times SP$

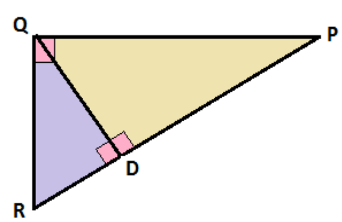
$p^2 = \frac{b^2}{c} \times \frac{a^2}{c} = \frac{a^2b^2}{c^2}$

$\Rightarrow p^2c^2 = a^2b^2$

$\Rightarrow pc = ab$

4) △ PQR, $\angle PQR = 90^0$, $QD \perp PR$,

If PD = 4DR , prove that PQ = 2QR.



PR = PD + DR ⇒ PR = 4DR + DR

PR = 5DR ---------- (1)

$QR^2 = PR \times DR \Rightarrow QR^2 = 5DR \times DR$ [∵from (1)]

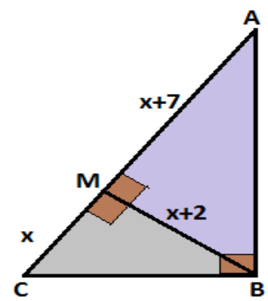
$QR^2 = 5DR^2 \Rightarrow QR = \sqrt{5}DR$ ---------(2)

$PQ^2 = PR \times PD \Rightarrow PQ^2 = 5DR \times 4DR \Rightarrow PQ^2 = 20DR^2$

$\Rightarrow PQ = 2\sqrt{5}DR$

$\Rightarrow PQ = 2QR$ [∵ from (2)]

5) ⊿ ABC, $\angle ABC = 90^0$, $BM \perp AC$,



(a) BM = x + 2, AM = x + 7, CM = x , find x

$BM^2 = AM \cdot CM$

$(x + 2)^2 = (x + 7)x$

$\Rightarrow x^2 + 4x + 4 = x^2 + 7x$

$\Rightarrow 4x + 4 = 7x$

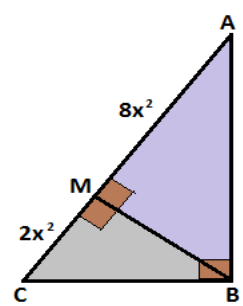
$\Rightarrow 3x = 4$

$\Rightarrow x = \frac{4}{3}$

(b). $AM = 8x^2$, $MC = 2x^2$ then, find BM and AB .

$BM^2 = AM.MC$

$BM^2 = 8x^2 . 2x^2 \Rightarrow BM^2 = 16x^4$



$\Rightarrow BM = \sqrt{16x^4}$

$\Rightarrow BM = 4x^2$

$AB^2 = AC \cdot AM$

$\Rightarrow AB^2 = 10x^2 . 8x^2 \Rightarrow AB^2 = 80x^4$

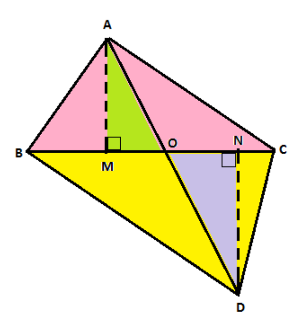
$\Rightarrow AB = \sqrt{80x^4}$

$\Rightarrow AB = 4x^2\sqrt{5}$

Exercise 11.5

1. ΔABC and ΔBDC are on the same base BC. Prove that

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DBC} = \frac{AO}{DO}$$



Soln : Draw $AM \perp BC$ and $DN \perp BC$.

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DBC} = \frac{\frac{1}{2} x BC x AM}{\frac{1}{2} x BC x DN} = \frac{AM}{DN} \text{ ------ (1)}$$

In ΔAOM and ΔDON

$\angle AMO = \angle DNO = 90^0$ [∵ construction]

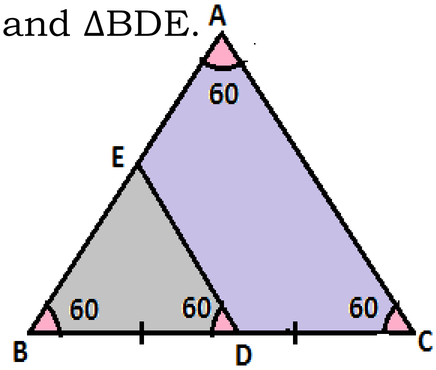
$\angle AOM = \angle DON$ [∵vertically opp angles]

$\therefore \ \Delta AOM \sim \Delta DON$

$\therefore \ \frac{AM}{DN} = \frac{AO}{DO} = \frac{OM}{ON}$

$\therefore \ \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DBC} = \frac{AO}{DO}$

2. ΔABC and ΔBDE are two equilateral triangles and BD = DC, find the ratio between areas of ΔABC and ΔBDE.



$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BDE} = \frac{BC^2}{BD^2}$

$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BDE} = \frac{(2BD)^2}{BD^2}$

$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BDE} = \frac{4BD^2}{BD^2}$

$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BDE} = \frac{4}{1}$

$\Rightarrow$Area of ΔABC: Area of ΔBDE = 4:1

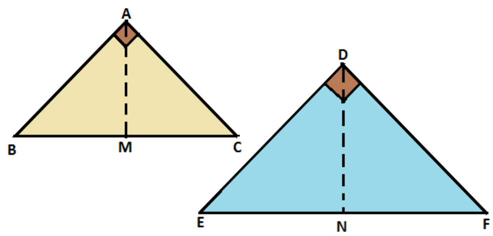
3. Two isosceles triangles are having equal vertical angles and their areas are in the ratio 9:16. Find the ratio of their corresponding altitudes.

Soln ; In ΔABM and ΔDEN,

$\angle AMB = \angle DNE = 90^0$ [∵ $AM \perp BC$, $DN \perp EF$]

$\angle ABM = \angle DEN = 45^0$ [∵in isosceles triangle remaining two angles are equal to 45^0]

$\therefore \ \Delta ABM \sim \Delta DEN$



$\therefore \ \frac{AB}{DE} = \frac{BM}{EN} = \frac{AM}{DN}$

$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AM^2}{DN^2} = \frac{3^2}{4^2}$

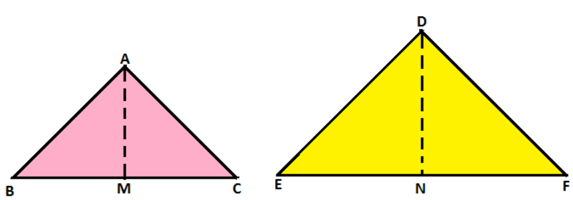
$\Rightarrow$ AM : DN = 3: 4

4. The corresponding altitudes of two similar triangles are 3cm and 5cm, respectively. Find the ratio between their areas .

$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AM^2}{DN^2}$

$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{3^2}{5^2}$

$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{9}{25}$



5. In the Trapezium ABCD, AB∥CD, AB = 2CD and area of ΔAOB=84cm² find the area of ΔCOD.

In ΔAOB and ΔCOD,

∠AOB = ∠COD [∵vertically opposite angles]

∠OAB = ∠OCD [∵AB∥CD alternate angles]

∴ ΔAOB ~ ΔCOD [∵AAA Similarity.]

$\therefore \frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta COD} = \frac{AB^2}{CD^2}$

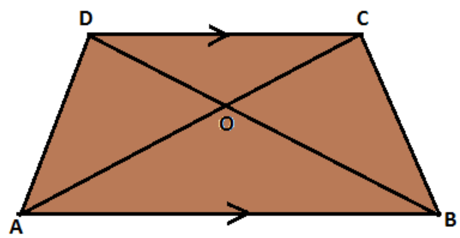
$\therefore \frac{84}{\text{Area of } \Delta COD} = \frac{(2CD)^2}{CD^2}$

$\therefore \frac{84}{\text{Area of } \Delta COD} = \frac{4CD^2}{CD^2}$

$\therefore \frac{84}{\text{Area of } \Delta COD} = \frac{4}{1}$

$\therefore \text{Area of } \Delta COD = \frac{84}{4}$

$\therefore \text{Area of } \Delta COD = 21cm^2$



6. In the above figure, find the ratios between areas o fΔAOB and ΔCOD, if AB = 3CD.

In ΔAOB and ΔCOD

∠AOB = ∠COD [∵vertically opposite angles]

∠OAB = ∠OCD [∵AB∥CD alternate angles]

∴ ΔAOB ~ ΔCOD [∵AAA Similarity.]

$\therefore \frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta COD} = \frac{AB^2}{CD^2}$

$\therefore \frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta COD} = \frac{(3CD)^2}{CD^2}$

$\therefore \frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta COD} = \frac{9CD^2}{CD^2}$

$\frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta COD} = \frac{9}{1}$

$\therefore \text{Area of } \Delta COD : \text{Area of } \Delta COD = 9 : 1$